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Hidden Markov Model and Financial Application

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Hidden Markov Model and
Financial Application

by

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Report

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Abstract

Hidden Markov Model and Financial Application

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A Hidden Markov model (HMM) is a statistical model in which the system being modeled is assumed to be a Markov process with numerous unobserved (hidden) states. This report applies HMM to financial time series data to explore the underlying regimes that can be predicted by the model. These underlying regimes can be used as an important signal of market environments and used as guidance by investors to adjust their portfolio to maximize the performance.

This report is composed of three chapters. The 1st chapter will introduce the difficulties in predicting financial time series, the limitations with traditional time series models, justification for choosing HMM and previous studies. The 2nd chapter will go through a detailed overview of HMM model, including the basic math frame works, and fundamental questions and algorithm to be addressed by the model. In the 3rd chapter, the trend analysis of the stock market is found using Hidden Markov Model. For a given observation sequence, the hidden sequence of states and their corresponding probability values are found. This analysis builds a platform for investors to decision makers to make decisions on the basis of probability and pattern of transition of each hidden state which cannot be observed from market data.

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Chapter 1: Introduction

A hidden Markov model (HMM) is a statistical model in which the system being modeled is assumed to be a Markov process with numerous unobserved (hidden) states. It is this modeling technique that we apply to predicting financial time series. This chapter will focus on the following several issues: characteristics of time series, difficulties in predicting financial time series, the limitations with traditional time series models, justification for choosing HMM and previous studies with HMM.

1.1 Financial Time Series

Financial time series are defined as a sequence of price movement over time. This report focuses on the US stock market because its volatile nature makes it challenging to data modelers. The data set used in this analysis is daily Standard & Poor 500 Index (S&P 500) starting from 1995 to 2006 which is long enough to cover a complete set of bull, bear and sideway market. Data is downloaded from Yahoo! Finance. It is a combination of weighed stock price of 500 US big companies from all sectors and it can best reflect the overall market healthiness.

Different transformations can be made to the original data set in order to achieve better model performance. The most common transformation made to the original time series data is calculating the return of time series. Details of data transformation will be illustrated in Chapter 3.

The task of modeling is develop a mathematical model to simulate the data series and then based on the mathematic model apply existing data sets to predict the state of future events. Generating a signal of probabilities more likely towards up or down, high volatile or low volatile as a signal for next trading day will be extremely important for investors and traders.

1.2 Limitations with Time Series Models

Most traditional time series models assume time series events follow a linear pattern by time and therefore they can be predicted by a linear auto regressive model in a form of $y_t = a + b \times y_{t-1} + \text{noise}$, where a and b are regression weights. Obviously, this method only works for a linear data set. But in reality, the market is affected by many different factors from all different sources, and these factors are changing so fast. Therefore stock prices are very likely independent of past prices, and are seen as a rapid change in data patterns. For different time windows, they show different patterns and therefore cannot be described with one stationary model and no single model can make accurate predictions all the time.

1.3 Market Technical Indicators

The nonlinear nature of financial time series makes it hard to predict for the future events. In order to explore more information carried by the data price, different transformations resulting in different technical indicators can be valuable. A number of technical indicators are extensively used among market indicators investors and especially traders whose trading decisions are based on stock price action. Common technical indicators include Moving Average Convergence Divergence (MACD), Price by Volume, Relative Strength Index (RSI), Average True Range (ATR) and many others. Each indicator is created based on a specific algorithm and therefore reflects only a specific aspect of information carried by the data. The analysis that follows uses ATR with details provided in Chapter 3.

1.4 Markov Process and Hidden Markov Model

A Markov Process is a system that has N states and the probability of being in a state at a particular time depends on a particular number of previous time states, but not the entire history. A first order Markov chain is a sequence of events where given the present point, the future is independent of the past. A HMM is a form of probabilistic finite state systems where the actual states are not directly observable. They can only be estimated using observable outcomes associated with the hidden states. At each time point, the HMM emits a symbol and changes a state with certain probability. HMM can be used to analyze and predict time series or time depending phenomena. There is not a one to one correspondence between the states and the observation symbols. Many states are mapped to one symbol and vice-versa.

Time Series Analyses can apply a Hidden Markov Model (HMM) with a Gaussian mixture at each state as the forecast generator. Therefore, it can model more complex series that do not fall into a single Gaussian distribution. The parameter estimates of the model can be updated at each iteration by using an Expectation-Maximization (EM) algorithm. At each time point, the parameters of the Gaussian mixtures, the weight of each Gaussian component contained in the output, and the transformation matrix are all updated in a dynamic fashion. The dynamic nature of the HMM model provides a good basis to model non-stationary financial time series data.

1.5 Previous Literatures

Many forecasting methods have been proposed and implemented for the stock market analysis. In this section we will review some previous studies that applied HMM.

Hidden Markov Model was first invented in speech recognition [9, 10], but is widely applied to forecast stock market data. Other statistical tools are used to make forecasts based on past time series data. Box–Jenkins used Time series analysis for forecasting and control[3]. White used Neural Networks for stock market forecasting and alternative learning algorithms and prediction methods were also tried[16,17]. Chiang et al. have used neural network to forecast the end-of-year net asset value of mutual funds [4]. Kim and Han showed that the complex dimensionality and buried noise of the stock market data makes it difficult to re-estimate the parameters of neural network[7]. Romahi and Shen showed that ANN occasionally suffers from over fitting problem [14]. They developed an evolving rule based expert systems and obtained a method which is used to forecast financial market behavior. Model hybridizations were also effectively used to forecast financial behavior. The drawback was the requirement of expert knowledge.

Much work has been also carried out with mortified techniques and algorithms for training models for forecasting or predicting the next day close value of the stock market, for which randomly generated transition probability matrices, emission probability matrices and prior probability matrices have been considered. To improve the prediction accuracy and handle overfitting problem, Hassan and Nath used HMM to achieve better optimization [11]. Hassan et al. proposed a mix model of HMM and neural network for stock Market forecasting, and he also combined HMM and fuzzy logic rules to improve the prediction accuracy on non-stationary stock data sets [12]. Jyoti Badge used technical indicators as an input variable instead of stock prices for analysis[1].

1.6 Proposed Analysis

The following uses a Hidden Markov Expert Model to forecast the change of the S&P 500 Index at a month interval. The underlying state, treated as "experts" for each regime which are usually invisible to the investor, can determine the behavior of the stock value,. These hidden states are derived from the emitted symbols. The emission probability depends on the current state of the HMM. Probability and Hidden Markov Model give a way of dealing with uncertainty. The analysis will focus on characterizing and predicting the regime for each trading day. Although the work doesn't provide a direct prediction of daily price movement, yet prediction of the class of regime is also important as a signal of market and can be utilized by traders and investors.

Chapter 2: Hidden Markov Model

Hidden Markov Models (HMMs) have been applied in many areas. They are widely used in speech recognition[6], bioinformatics [2], and semiconductor malfunction [15]. This chapter will describe the math framework of Hidden Markov Model (HMM), three fundamental problems and the algorithm used to solve each of the problems.

2.1 Overview of HMM

A Hidden Markov Model is a tool for representing probability distributions over sequence of observations. The Hidden Markov Model is named for two properties: first, it assumes that the observation at time t is generated by a process whose state St is hidden from the observer; second, it assumes that the state of this hidden process satisfies the Markov property, that is, given the value S_{t-1}, the current state S_t only depends on the state S_{t-1}, and is independent of all the states prior to time t-1.

The general HMM approach framework is an unsupervised learning technique which allows us to find new patterns without having to impose a template during the learning process.

The financial time series is generated by some underlying stochastic process that is most likely associated with market environment and investment decisions unknown to the public. Therefore, there is a good match between sequential data and HMM in which the prediction for the next state only depends on the current state not on the whole history of the past process. A HMM model can be treated as an imaginary professional with the hidden power that drives the financial market up and down.

2.2 Elements of HMM

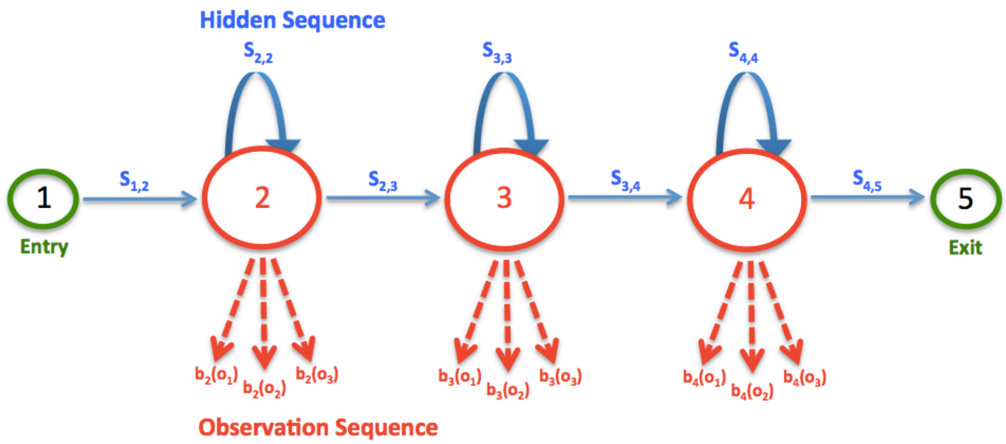

Figure 1-1. Diagram of HMM. It includes a sequence of observations indicated by O and a sequence of hidden states indicated by S. Sij is the probability of moving from one state to another.

The structure of the Hidden Marcov Model is illustrated in Fig 1-1. An HMM consists of the following elements:

(1) A set of hidden or latent states (S)

(2) A set of possible output symbols (O)

(3) A state transition probability matrix (A), probability of making transition from one state to each of the other states $A = \{a_{ij}\}$ where

$$a_{ij} = P(q_{t+1} = S_j \,|q_t = S_i), \;\; 1 \leq i, j \leq N$$

(4) Observation emission probability matrix (B), probability of emitting/observing a symbol at a particular state, The observation probability distribution in state $j, B = \{b_j(k)\}$where

$$b_j = P(v_k \; at \;\; t \,|q_t = S_j), \;\; 1 \leq j \; \leq N, 1 \leq k \; \leq M$$

(5) The prior probability $\pi_i = \{\pi_i\}$ of being in state i at the beginning of the observations where

$$\pi_i = P(q_1 = S_i), \ 1 \leq i \leq N$$

To initiate an HMM, an initial state will be chosen based on the prior distribution π and t is set at 1. $O_t = v_k$ is chosen according to the distribution in S_i. The model moves to state $q_{t+1} = S_j$ based on the transition probability distribution of S_i. This process will continue as t increments or until termination. More formally, this process is denoted by:

λ=(S, O, A, B, π) where

S={s_1,s_2,...,s_N} is a set of N possible states

O={o_1,o_2,...,o_M} is a set of M possible observation symbols

A is an NxN state Transition Probability Matrix (TPM)

B is an NxM observation or Emission Probability Matrix (EPM)

π is an N dimensional initial state probability distribution vector and A, B and π where

$$\sum_j a_{ij} = 1, \quad \sum_t b_i(O_t) = 1, \quad \sum_i \pi_i = 1, \quad a_{ij}, b_i(O_t), \pi_i \geq 0 \text{ for all } i, j, t$$

2.3 Fundamental Problems with HMM

There are three fundamental issues regarding HMMs that must be solved before an HMM can be used.

Given $\lambda = (A, B, \pi)$ and observation sequence O = O_1, O_2, O_3, . . . , O_T, how can $P(O|\lambda)$ be computed for each observation sequence?

Given $\lambda = (A, B, \pi)$ and observation sequence O = O_1, O_2, O_3, . . . , O_T, what is the underlying state sequence that has the highest conditional probability?

Given an observation sequence O = $O_1, O_2, O_3, \ldots, O_T$ and a number of possible models, how do we maximize $P(O|\lambda)$ by adjusting model parameters $A, B,$ and π?

2.4 Forward-backward algorithm

The 1st question can be solved by calculating the probability that model has generated sequence O by using Forward-Backward algorithm [4, 5]. The Forward-Backward algorithm uses induction to create two sets of probabilities, the forward and backward. Together, these are used to create a smoothed set of values.

Let λ = (A, B, π) be the full set of parameters. Given λ, we address the question of how to calculate efficiently P(O|λ), which is the probability of some given sequence of observed outputs. We consider an efficient method of calculating this by defining

$\alpha(t,i)=P(O = O_1, O_2, O_3, \ldots, O_T, qt = Si)$ (2.1)

This is a joint probability that the sequence of observations seen up to and including time t is $O_1,O_2,O_3,\ldots,O_t$, and that the state of the HMM at time t is S_i. The $\alpha(T,i)$ are called forward probabilities. The forward algorithm can be described in the following way:

- Initialization: $\alpha(1, i) = \pi ibi(O1)$
- Induction: $\alpha(t+1,i) = \sum_{j=1}^{N} \alpha(t,j)a_{ji}b_i(O_{t+1})$
- Termination: $P(0) = \sum_{i=1}^{N} \alpha(T,i)$

Unlike the forward algorithm in which we calculated $\alpha(t,i)$ successively forward in time. In the backward algorithm, we calculate another quantity but backward in time. The goal of the backward algorithm is to calculate the probability $\beta(t,i)$ defined by

$\beta(t, i) = P(O_{t+1}, O_{t+2}, \ldots, O_T | qt = Si)$, for $1 \le t \le T - 1$. (2.2)

For convenience, we set β(t, i) to 1, for all i. We then compute equation (2.2) working backwards from t = T − 1. The induction step of the procedure entails the equation:

$$\beta(t-1,i) = \sum_{j=1}^{N} a_{ij} b_j(O_t)\beta(t,j).$$

It may be shown that

$$P(q_t = k, q_{t+1} = l|O,\lambda) = \frac{\alpha(t,k)a_{kl}b_l(O_{t+1})\beta(t+1,l)}{P(O|\lambda)}.$$

2.5 Viterbi Algorithm

The second question is to calculate the most likely sequence of hidden states that produced this observation sequence O given a model and the observation sequence. The process is to decode the system by calculating the most likely sequence of hidden states that produced this observation sequence O.

This question becomes extremely important in speech recognition. Usually this problem is handled by Viterbi Algorithm [9, 10]. The Viterbi algorithm, based on dynamic programming, is used to find the most likely sequence of hidden states – called the Viterbi path – that results in a sequence of observed events. Viterbi is commonly used because it takes into account the most likely state at every instant and the probability of occurrence within the sequence of states. The algorithm will find the max Q for a given observation sequence O by the means of induction. An array δt(i) is used to store the highest probability paths, define:

δt(i) = max P({q1,q2,...,qt−1,qt =Si} ∩ { $O_1, O_2, O_3, \ldots, O_T$}) (2.1)

So, $\delta t(i)$ is the maximum probability of all possible ways to end up in state Si at time T having observed the sequence $O_1, O_2, O_3, \ldots, O_T$. Then

max $P(Q \cap O)$ = max δT (i)

To find a sequence Q for which the maximum conditional probability in equation (2.1) is achieved. Since

max $P(Q|O)$ = max $P(Q \cap O) / P(O)$

The denominator on the right-hand side does not depend on Q, so:

arg max $P(Q|O)$ = arg max $P(Q \cap O) / P(O)$ = argmax $P(Q \cap O)$

Hence, we have the algorithm as follows:

- Initialization step: $\delta 1(i) = \pi i b i(O1)$, $1 \leq i \leq N$.
- Induction step: $\delta t(j) = \max 1 \leq i \leq N\ \delta t-1(i) aij bj(Ot)$, $2 \leq t \leq T$, $1 \leq j \leq N$
- Update time: t=t+1 Return to step 2 if $t \leq T$ else terminate the algorithm

2.6 Baum-Welch algorithm

The third question is to determine HMM parameters $\lambda = \{A, B, \pi\}$ that best fit training data, given some training observation sequences $O = O_1, O_2, O_3, \ldots, O_T$ and general structure of HMM (numbers of hidden and visible states). The problem can be reformulated as find the parameters that maximize the following probability: argmax P $(O| \lambda)$.

There is no known analytic method to choose λ to maximize the probability. However, we can use a local maximization algorithm to find the highest probability. This algorithm is called the Baum-Welch [9, 10]. This is a special case of the Expectation Maximization method. It works iteratively to improve the likelihood of P $(O| \lambda)$. This iterative process is called the training of the model. The Baum-Welch algorithm is numerically stable with the likelihood non-decreasing of each iteration. It has linear convergence to a local optima.

To work out the optimal model $\mu = (A, B, \pi)$ iteratively, the Baum-Welch method uses the Forward Backward algorithm to create a re-estimation model of $\bar{\lambda} = (\bar{A}, \bar{B}, \bar{\pi})$. By using $\bar{\lambda}$ in place of λ, the probability of O being observed from the model can be increased up to some point. This can be defined by

$$\max_{\bar{\lambda}}[Q(\lambda, \bar{\lambda})] = P(O|\bar{\lambda}) \geq P(O|\lambda)$$

where the likelihood function converges to a critical point.

Given the above definitions we begin with an initial model λ and run the training data O through the current model to estimate the expectations of each model parameter. Then we can change the model to maximize the values of the paths that are used. By repeating this process we hope to converge on the optimal values for the model parameters.

2.7 Summary

In this chapter, the general form of HMM is introduced. We also studied the three basic problems involved with any HMM. A specific model design based on a specific data set is based on the basic model and the techniques to solve the three problems. In reality, in the prediction task, both Viterbi algorithm and the Baum-Welch algorithm are used in combination to make the system work.

Chapter 3: Analysis of SP500 Index with HMM

This chapter will construct HMM models based on Standard & Poor 500 Index (S&P500) from 1950 to 2016 as an application of HMM model to financial time series. Since it's hard to directly predict price movement for financial time series data due to its volatility nature, we will instead focus on the underlying state, assuming there exists an expert at each state of the Hidden Markov Model. We will do it in two ways. First, we will explore the bull/bear market transition. We can clearly see the importance of finding the switch between bull and bear market. Second, we will explore the transition between different volatility states. We assume that one of the experts is good at forecasting low volatility data, the second expert is best for high volatility regions, and the 3rd one for middle volatility regions. Computing the posterior odds being at each state for each time point provides a good guidance about market environment.

3.1 Data Set

The data set used in my analysis is Standard & Poor 500 Index (S&P 500 index) starting from beginning 1950 to 2016, downloaded from Yahoo Finance. It is a combination of weighed stock price of 500 US big companies from all sectors and it can best reflect the overall market healthiness and therefore are extensively studied.

The market price for each day is calculated as a daily index, which is composed of an opening price, a closing price, the highest price during the day and the lowest price during the day. For this analysis, we used the closing price at the last trading day of each month as the index of monthly price. The market closing price is commonly used by market traders as an indication of the price action of a day, a week or a month. The overall trend from 1950 to 2016 is shown in Fig 3-1. It can be divided into 3 periods. The market price gradually increases before 2000. It's getting choppy between 2000 and 2010. After 2010, it starts another upward trend.

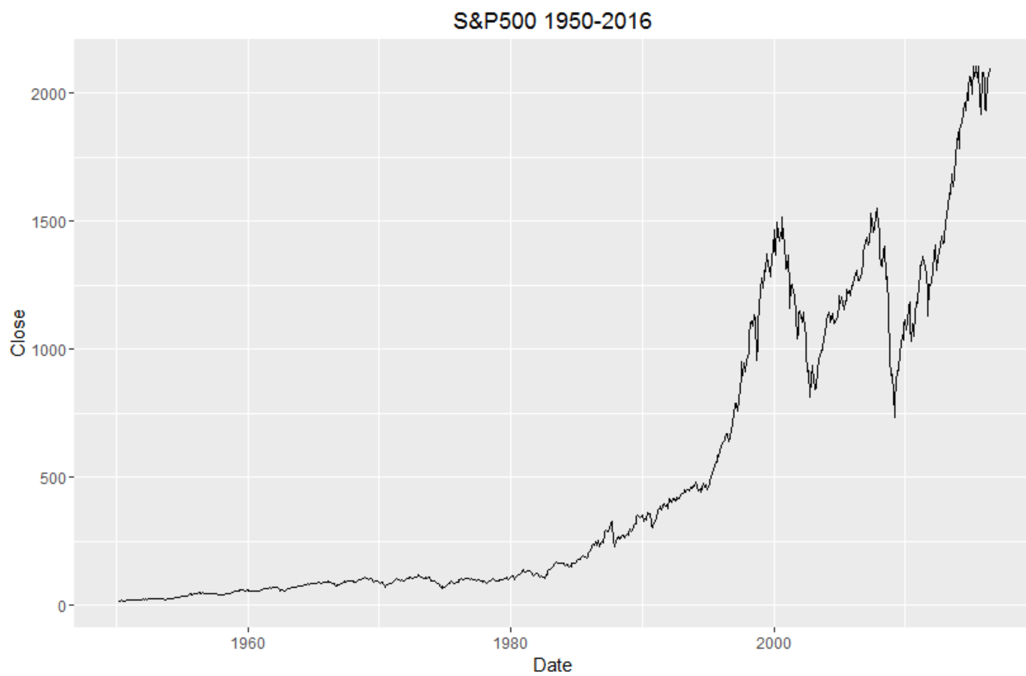

Figure 3-1. Trend of S&P 500 index between 1995 and 2016. Monthly closing prices are plotting in time.

A common assumption in many time series models is that data sets are stationary. A stationary process has the property that the mean, variance and autocorrelation structure do not change over time. Stationarity can be defined in precise mathematical terms, but for our purpose, we mean a flat looking series, without trend, constant variance over time, a constant autocorrelation structure over time and no periodic fluctuations. One way to check if data is stationary is by calculating autocorrelation (ACF). It is the correlation between two variables under the assumption that we know and take into account the values of some other set of variables. ACF based on original SP500 index shows that data is not stationary (Fig 3-3, top).

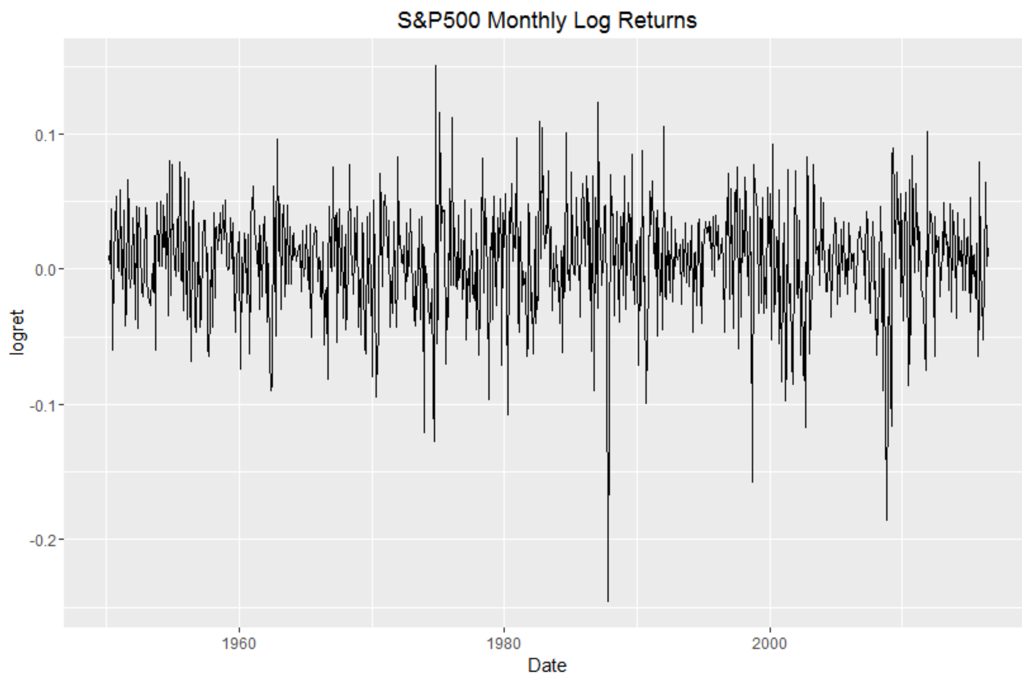

Figure 3-2. Log return of S&P 500 index. Detailed calculations can be found in text.

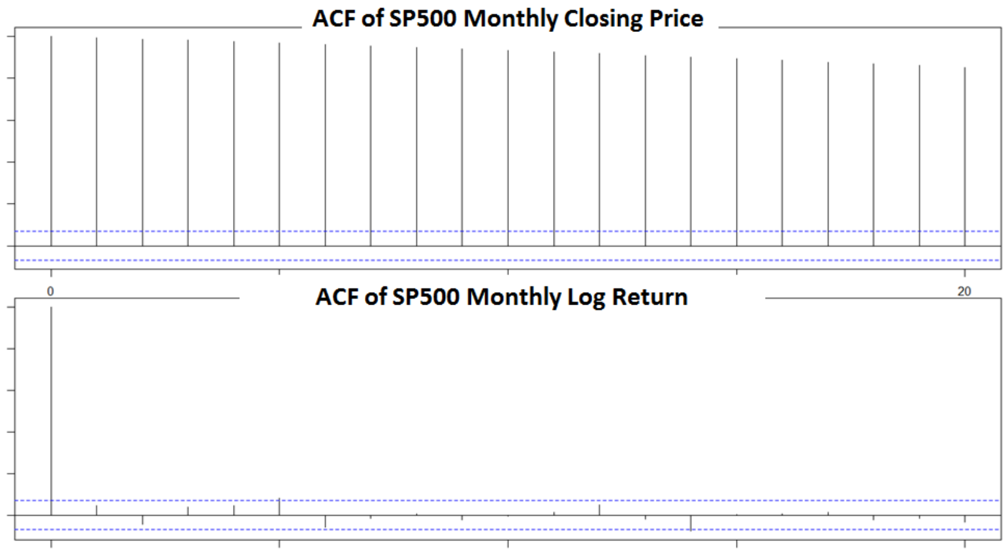

Figure 3-3. ACF. ACF based on S&P 500 monthly price (top) and ACF based on S&P 500 monthly log return (bottom).

In order to better model the data set, different transformations can be made to the original data set because modeling works better on stationary data set. The most common transformation made to the original time series data is calculating the return of time series. The calculation of return is following: $R_t = (P_t – P_{t-1}) / P_{t-1}$ (R_t : return at time t, P_t : price at time, P_{t-1} : price at time t-1). Based on calculating return, Log return and square return are often used to achieve stationary data set. In my analysis, I calculated Log return (Fig 3-2) and ACF measured after transformation showed a much better stationarity (Fig 3-3, bottom). It justifies why we transform original SP500 index and use the transformed data to build our models.

3.2 Identify Bull/Bear State Switch with HMM Model

Markov Models are a probabilistic process that looks at the current state to predict the probability moving to the next state. Hidden Markov Model (HMM) comes into play when the complexity lies in not knowing the probability of each regime shift and how to account for these probabilities changing over time. They are able to estimate the transition probabilities for each regime and then, based on current conditions, output the most probable regime.

In this analysis, we apply Hidden Markov Expert Model to forecast the underlying state change of the S&P 500 Index at a month interval. We define the regimes as being two states, bull market and bear market. Of course we can also extend to three state, bull, bear, or sideway. Here we only use two states as an example to show the application we can use HMM onto financial data. Our goal is to identity the switch between these two states and to identify a pattern of the two states. The entire analysis was coded in R and depmixS4 library was used for modeling and ggplot2 library was used for plotting.

The summary (Fig 3-4) below shows AIC and BIC for the model. AIC and BIC measures how much information was captured by the model. It's useful when we have more than one model to choose from. The transition matrix in Fig 3-4 tells us the probability of moving from one state to each of the states. We can see that the initial state is in state 2. Under this current state, the transition matrix tells us that there is a 96.3% chance that it stays in state 2 and there is 3.7% chance it moves to state 1, based on the current data set. Since we only have one response variable that is S&P 500 Log returns, we only have one Gaussian mixture model, as shown in summary.

```
Convergence info: Log likelihood converged to within tol. (relative change)
'log Lik.' 1440.542 (df=7)
AIC:  -2867.083
BIC:  -2834.326

Initial state probabilties model
pr1 pr2
  0   1

Transition matrix
        toS1  toS2
fromS1 0.837 0.163
fromS2 0.037 0.963

Response parameters
Resp 1 : gaussian
    Re1.(Intercept) Re1.sd
St1          -0.013  0.064
St2           0.010  0.033
```

Figure 3-4. Summary of HMM model based on S&P 500 Log returns. Information about the initial state, transition matrix, and distribution of each response variables is shown.

To examine the posterior probabilities of being in each state at each time point for a given sequence of observations and a given Hidden Markov Model, we calculated posterior odds over the entire data set. Fig 3-5 shows posterior odds for head and tails of the data set. From time point 1-6, there is a higher probability staying at S2 than S1, same with time point 791-796. It's not surprising because S2 stands for bull market and the US stock market has a prominent feature of long bull period and short bear period which can easily observed from Fig 3-1. Next we see the transition from bull to bear market happens over some periods.

	state	S1	S2
1	2	0.00000000	1.0000000
2	2	0.01941237	0.9805876
3	2	0.02024584	0.9797542
4	2	0.02226685	0.9777332
5	2	0.12564838	0.8743516
6	2	0.05794576	0.9420542

	state	S1	S2
791	2	0.05987022	0.9401298
792	2	0.12283074	0.8771693
793	2	0.06456177	0.9354382
794	2	0.05228596	0.9477140
795	2	0.02425900	0.9757410
796	2	0.01812486	0.9818751

Figure 3-5. Posterior probability of being in state S1 and S2 for head and tail of the data set. State indicated the preferred state of each time point by comparing the probability being each of two states S1 and S2.

Fig 3-6 shows the posterior odds over the entire data set. The 1st graph on the top is price over time. The 2nd plot shows the Log Return calculated based on original price. It's randomly distributed around 0 and can't be predicted. The 3rd plot is the probability being in Bear state at each time point. The bottom plot is the probability being in Bull state at each time point. From the two bottom plots, we can clearly see the transition between bull/bear states. Further, we can see the transition is in a neat cyclic pattern (rectangle in red) in time that cannot be directly observed from original price chart or log returns. When bear state takes over, it corresponds to a sizable correction. This correlation can be clearly observed from the zoom in graph in Fig 3-7 which only focus on a short period covering the 1st few years and the 1st bear control (blue arrows on both Fig 3-6 and 3-7). It suggests the importance of this model. If we cannot accurately predict the index price as most time series model do, and neither find a regular pattern to predict the trend for future events, then we can look at the transition between two or even more different underlying states which may show a preferred pattern and can be used indirectly as a prediction to the market.

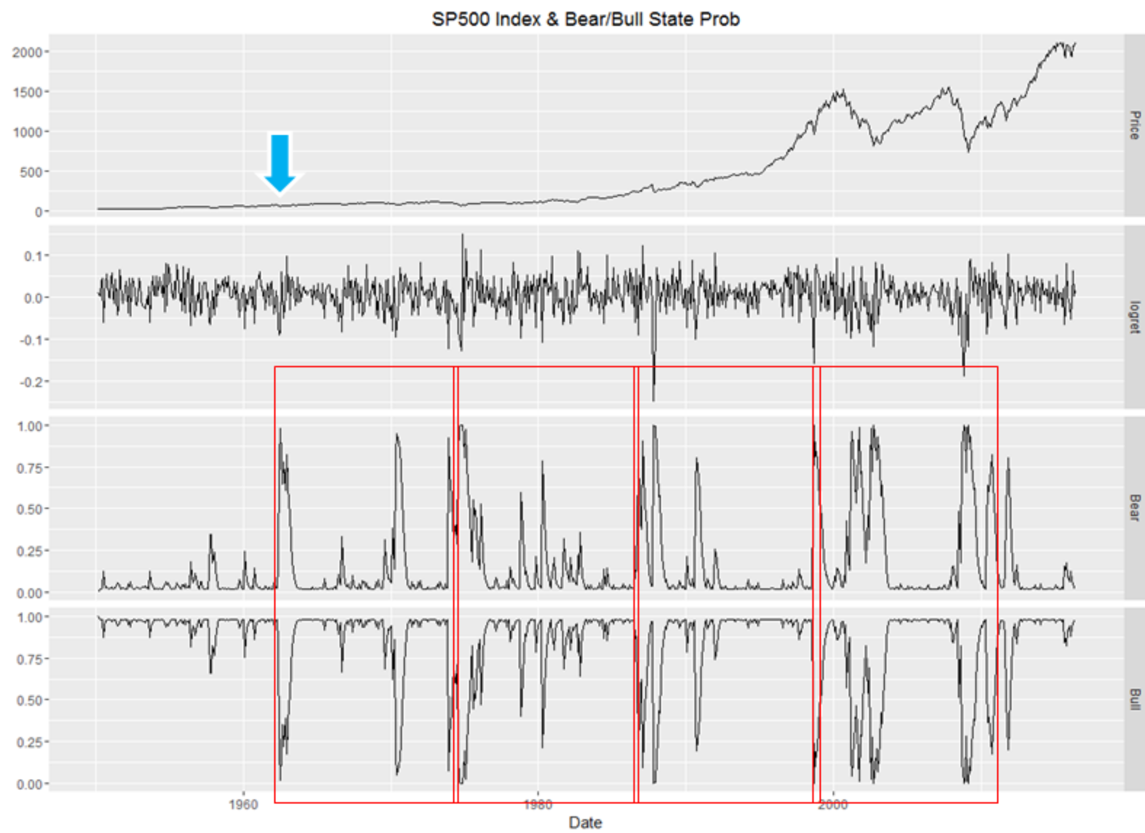

Figure 3-6. Bull/Bear state transitions over the entire time. Top two plots are the S&P 500 price and transformed Log returns. Bottom two plots are the probability staying in bear state and bull state at each time point. Red rectangle indicates the cyclic pattern over time for both bull and bear states.

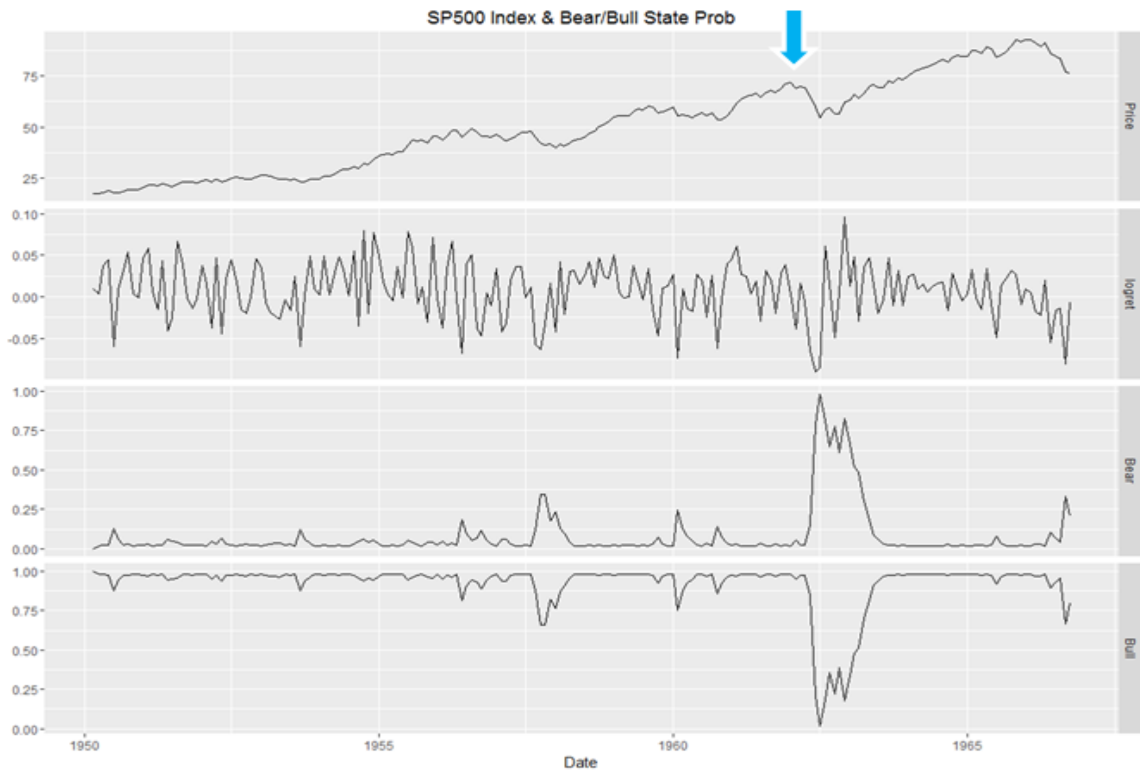

Figure 3-7. Bull/Bear state transitions from 1950-1967. It shows when a bear state in control can cause a size market correction. Blue arrow indicates the correction that is also indicated in Fig 3-7 but difficult to be seen.

3.3 Average True Range (ATR)

In order to explore more information carried by the data price, different transformations are often made to explore the underlying patterns that are not easily observed from original data point. For financial time series, a number of technical indicators are come up by different transformations and they are extensively used among market indicators investors and especially traders because whose trading decisions are mostly made based on stock price action and less depend on the financial fundamentals of company. They believe that even under the situation when the company fundamentals do not change, the price movement can be large enough for them to trade. A number of common technical indicators are Moving Average Convergence Divergence (MACD), Price by Volume, Relative Strength Index (RSI), Average True Range (ATR) and many others. Each indicator is created based on a specific algorithm and therefore reflects only a specific feature of the data. We will use ATR through my analysis. ATR measures a security's volatility. High ATR values indicate high volatility and may be an indication of panic selling or panic buying. Low ATR readings indicate sideways movement by the stock.

The Average True Range (ATR) is based on 14 periods and is calculated on a daily basis. Because there must be a beginning, the first ATR value is simply the High minus the Low and the first 14-day ATR is the average of the daily ATR values for the last 14 days.

3.4 Importance of Market Volatility

Investors general success hinges on long-term thinking. However, most of them can’t help worrying about short time adjustment with their portfolios. This worry is caused by recent increases in volatility over the last few years. History has shown that the stock market and the economy move in cycles that repeat over and over. Therefore, understanding the different stages of the economy can help guide investment decisions.

In a bull market, where investors are showing immense confidence, they tend to raise stock holding position. Conversely, safe-haven assets, like gold and bonds, will fall by the wayside. In a bear market, which indicates a lack of confidence in the economy, investors usually turn toward safe havens assets, adjusting the percentage of bonds upward. Bonds are less likely to lose money than stocks are and can reduce your portfolio's losses during stock market declines.

Obviously, knowing different market conditions can affect the investment strategy and therefore have a huge impact on the investment returns. Figuring out when to start, stop or change a trading strategy, adjusting risk and money management techniques, and even setting the parameters of entry and exit conditions are all dependent on the market conditions or so called “regimes”.

Being able to identify different market regimes and altering your strategy accordingly can mean the difference between success and failure in the markets. In this study, we will explore how to identify different market regimes by using a powerful class of machine-learning algorithms known as Hidden Markov Models.

3.5 Identify High/Mid/Low Volatility State Shift with HMM Model

In this analysis, ATR was calculated and both Log Return and ATR were used as response variables to construct our model. This model has two Gaussian mixture models at each time. To **i**dentify High/Mid/Low volatility state change, we define the regimes as being different volatility, which is an important factor and having a large effect on the performance of trading strategy. The “experts” at each state of the Hidden Markov Model applied for financial time series prediction are three regimes, with characteristics and predicting bias distinct from each other. One of the experts is good at forecasting low volatility data, the second expert is best for middle volatility regions, and the third one is good for high volatility data. We are looking to find different market regimes based on these factors that we can then use to optimize trading strategy.

The model summary is shown in Fig 3-8. The transition matrix gives us the probability of moving from one state to the next. From the transition matrix, the initial state probabilities model is in state 1. Under this current state, we can see from the transition matrix that there is a 99.8% chance that it moves from S1 to S1, 0.02% chance it moves to state S3.

```
Convergence info: Log likelihood converged to within tol. (relative change)
'log Lik.' -622.8236 (df=20)
AIC:  1285.647
BIC:  1378.884

Initial state probabilties model
pr1 pr2 pr3
  1   0   0

Transition matrix
        toS1  toS2  toS3
fromS1 0.998 0.000 0.002
fromS2 0.000 1.000 0.000
fromS3 0.000 0.008 0.992

Response parameters
Resp 1 : gaussian
Resp 2 : gaussian
    Re1.(Intercept) Re1.sd Re2.(Intercept) Re2.sd
St1           0.006  0.039           3.057  1.608
St2           0.005  0.045          49.964 12.902
St3           0.009  0.043          12.584  1.853
```

Figure 3-8. Summary of HMM model using both Log returns and ATR as response variables.

Fig 3-9 below shows the 3 regime shift over the entire time. We can see before 1998, low Volatility state has been in control (4th plot labelled as “low V”). Then mid Volatility state kicks in (blue arrow) and then gradually shift to high Volatility state (green arrow).

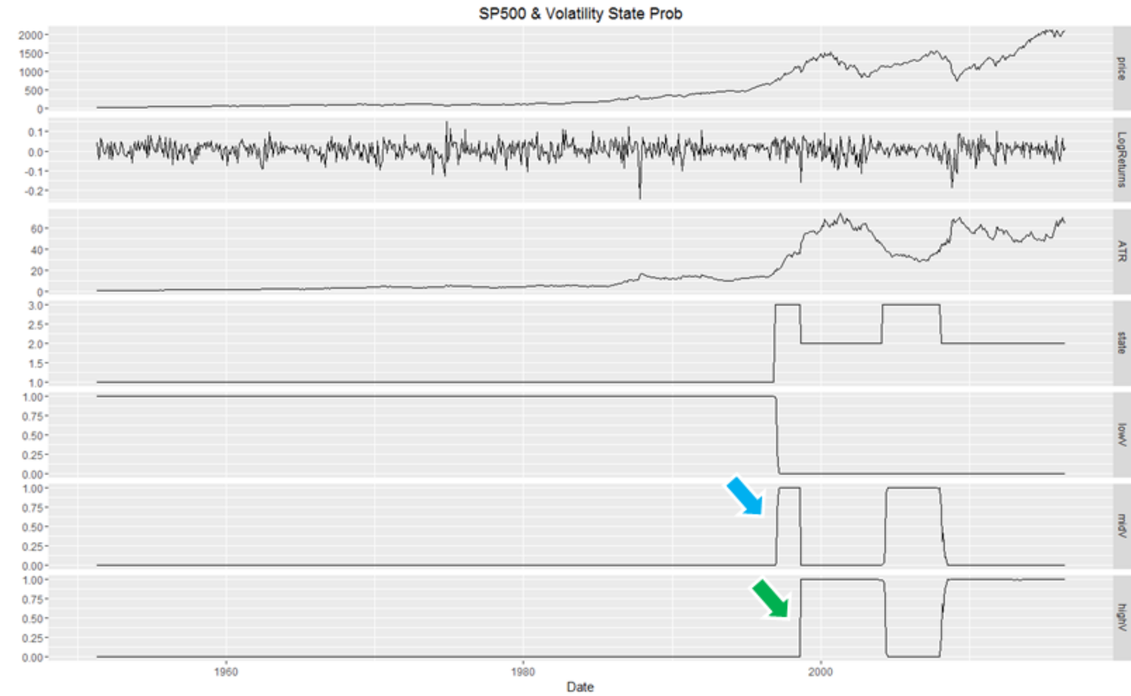

Figure 3-9. Regime shift over the entire time series. From top to bottom, the plots are: S&P 500 index price, Log returns, ATRs, preferred state, probability of low volatility, middle volatility and high volatility at each time point. The blue arrow indicates the shift from low volatility to middle volatility. The green arrow indicates the shift from middle volatility from high volatility.

3.6 Summary

This report showed two examples of HMM model and value of studying market environment. However, unlike other time series model in which future events can be predicted based on recent events, HMM model can't predict the future market price based on history. But it can reveal more underlying information other than index price. The underlying states can indirectly guide the direction of market price and therefore are equally important. For example, the bear/bull state show a more clear temporal pattern than the original market price based on this analysis (Fig 3-6). It's clear that every 10 years the market experiences a transition between two states. Based on this temporal pattern, it reasonable to expect that the next big bull/bear transition will happen around

~year 2020. This is the predict of the martket based on the model. However, at this statge, since test data in not available, this conlusion cannot be validated.

In the future, model should be implemented on big data platform to allow in streaming analytics. Based on this platform, models can be updated in a real time fashion and the trading strategy can be adjusted quickly to survive in a volatile market. The model is constructed based on most recent data. By looking at the current regime, the current market can be future investagated and the change of market regime will provide a shift of market environment. This is not a real prediction for the future event. However, by real time updating the model, it can give us the better update of the market regime.

References

[1] J. Badge, “Forecasting of Indian Stock Market by Effective Macro- Economic Factors and Stochastic Model,” Journal of Statistical and Econometric Methods, V ol. **1** (2),pp. 39-51,ISSN: 2241-0384 (print), 2241-0376 (online) Sciencepress Ltd, 2012.

[2] P. Baldi, Y.Chauvin, T.Hunkapiller, and M. A. McClure. Hidden Markov Models for biological primary sequence information. Proc.Nat.Acad.Sci. (USA), 91(3): 1059-1063, 1994.

[3] G. E. P. Box and G. M. Jenkins, Time series analysis: forecasting and control. San Fransisco, CA: Holden- Day, 1976.

[4] W. C. Chiang, T. L. Urban and G. W. Baldridge,“A neural network approach to mutual fund net asset value forecasting,” Omega International Journal of Management Science.,Vol. **24** (2), pp. 205–215,1996.

[5] A. Gupta and B. Dhingra, Non-Student members, IEEE, “Stock Market Prediction Using Hidden Markov Models,” 2012.

[6] B. H. Juang and L.R. Rabiner. Hidden markov models for speech recognition. Technometrics, 33:251-172, 1991.

[7] K. J. Kim and I. Han, “Genetic algorithms approach to feature discretization in artificial neural networks for the prediction of stock price index, Expert Systems with Applications,” Vol.19,pp. 125–132,2000.

[8] H. M. K. Mok, “Causality of interest rate, exchange rate and stock prices at stock market open and close in Hong Kong,” Asia Pacific Journal of Management,Vol. **10** (2), pp. 123–143,1993.

[9] L. Rabiner and B. Juang, “Fundamentals of Speech Recognition,” Prentice-Hall, Englewood Cliffs,NJ, 1993.

[10] L. R. Rabiner, “A tutorial on HMM and Selected Applications in Speech Recognition,” In:[WL], proceedings of the IEEE,Vol. **77** (2), pp. 267- 296,1993.

[11] R. Hassan and B. Nath, “Stock Market forecasting using Hidden Markov Model: A New Approach,” Proceeding of the 2005 5 international conference on intelligent Systems Design and Application 0-7695-2286-06/05, IEEE, 2005.

[12] Md. Rafiul Hassan, Baikunth Nath and Michael Kirley, “ A fusion model of HMM, ANN and GA for stock market forecasting,” Expert systems with Applications., pp. 171-180,2007.

[13] Hassan Rafiul, Nath Baikunth and Michael Kirley, “HMM based Fuzzy Model for Time Series Prediction,” IEEE International Conference on Fuzzy Systems., pp. 2120-2126,2006.

[14] Y. Romahi and Q. Shen. “Dynamic financial forecasting with automatically induced fuzzy associations,” In Proceedings of the 9th international conference on fuzzy systems., pp. 493–498,2000.

[15] P. Smyth. Hidden Markov models for fault detection in dynamic systems. Pattern Recognition, 27(1):149-164, 1994.

[16] H. White, “Economic prediction using neural networks: the case of IBM daily stock returns,” In Proceedings of the second IEEE annual conference on neural networks., II, pp. 451–458,1988.

[17] H. White, Learning in artificial neural networks: a statistical perspective, Neural Computation., Vol. **1** , pp. 425–464,1989.